1. Introduction

The large scale structure of the earth is caused by *geodynamic processes* which are explained using *energetic*, *kinematic* and *dynamic* descriptions. While "geodynamic processes" are understood to include a large variety of *processes* and the term is used quite loosely, the methods of their *description* involve well defined fields. *Energetic* descriptions are involved with distribution of energy in our planet, typically expressed in terms of heat and temperature. *Kinematic* descriptions describe movements using velocities, strains and strain rates. *Dynamic* descriptions indicate how stresses and forces behave.

In the field, we document only the *consequences* of geological processes. The underlying *causes* are much harder to constrain directly. Nevertheless, if we want to explain the tectonic evolution of our planet, we need to interpret these causes or: *"driving forces"* . For this, we need to find a *dynamic description* of geological processes that is consistent with our observations. Our descriptions relate *causes* and *consequences – tectonic processes* with *field observations.* In many cases, we will use equations as a concise form to describe processes and observations in nature. As we will be dealing mostly with large scale tectonic questions, the observations that we shall use are also on a large scale. For example, we shall use observations on the elevation (Fig. 1.1) and heat flow of mountain ranges, the thickness of continents and the water depth of the oceans.

As the processes we seek to describe are *changing* with time or space, many of the descriptive equations will be *differential equations.* However, it is *not* the aim of this book to confuse the reader with high level mathematics. Rather it is the aim to introduce field geologists to the beauty and simplicity of descriptions with equations. Thus, all equations will be explained from basic principles so that an intuitive understanding can be achieved. While I prefer the term "description", many colleagues would refer to the contents of this book as "modeling". Thus, it appears useful to begin this book with a discussion what we mean when we use the word "model".

1.1 What is a Model?

Models are tools that we use to describe the world around us in a simplified way so that we can understand it better. Sadly, many geologists misunder-

stand the word model and think of it only as something complicatedly mathematical that has little to do with field work (s. Greenwood 1989). In doing so, many field geologists overlook the fact that field mapping itself is also a form of modelling:

Geological Mapping: A Typical Modeling Exercise. A map is a transformation of reference frames; for example the projection of the geographic position of field locations onto a piece of paper. However, a geological map that is a mere representation of field data in a new reference frame (i.e. our piece of paper) may be a good map but is usually a poor *geological* map (Passchier et al. 1990). It may still be useful to find a given outcrop of a given lithology, structure or metamorphic grade, but as scientists we are usually unsatisfied by simply documenting what is there. Usually we want to go beyond that and map to clarify field relationships, for example when we infer where a lithological contact is underneath a meadow. We make maps so that we can explain some features of nature to a colleague geologist without him or her having to do the same amount of field work we invested to produce the map in the first place. In order to achieve this aim, the geological map must illustrate field relationships in a *simplified* and *interpreted* manner. This forces the field geologist to a constant decision-making process. First of all the geologist has to decide *what* is to be mapped. Is it topography? Is it structure? Is it metamorphic isograds or is it lithology? Which of these (and many others) is to be mapped depends on the question with which we go into the field (Fig. 1.1). Then, the geologist has to decide on the *scale* on which the map should be produced. This decision is not trivial! The scale of the map depends on the problem to be solved. Once the scale is decided upon many more decisions are to be made. Which observations are too small to be mapped and should be neglected? Which ones should be drawn into the map? Which ones are to be emphasized by lines? Can a contact seen in two outcrops be mapped as a line, even in the paddock separating the two outcrops? The geologist is modeling!

If the map is good, then it helps the reader - like any other good model - to understand nature quickly and easily. It also helps to make predictions how the geology may look at different places that were not mapped yet. For example, constructing profiles across our map helps us (to a certain degree) to explain how the geology looks underground. In numerical-, analogue-, conceptual- or thought-models this process is identical. Mathematical models consist of a series of rules that determine which observations in nature are to be neglected and which ones are to be emphasized. The former will *not* appear in an equation, the latter *will* appear as a parameter in the equation. As such, a mathematical model is no different from the field work of a geologist.

Every model can be considered as a tool that can be used to make predictions about observations in nature. Just as a geological map can be used to construct cross sections and thus predict the geology underneath the surface, a numerical model can be used to make predictions about temperatures,

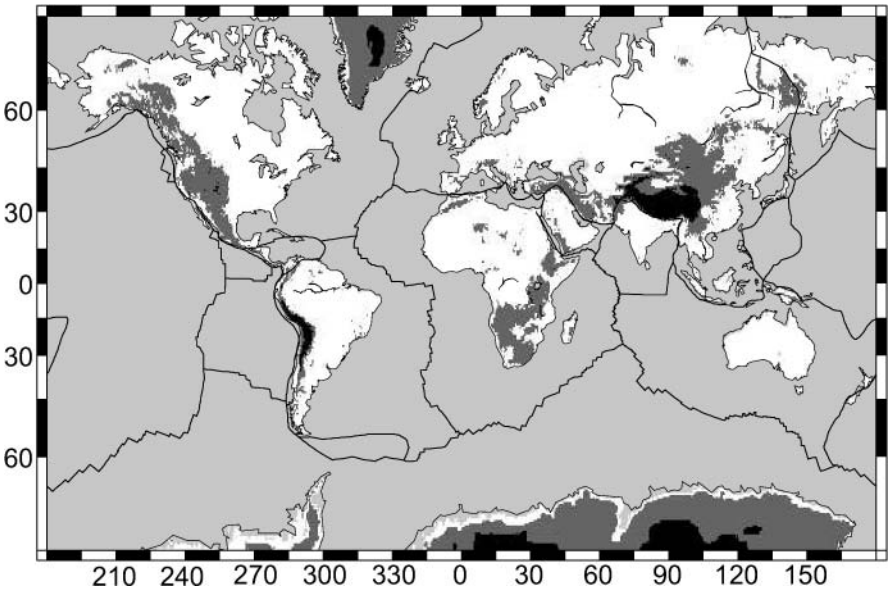


Figure 1.1. Topographic map of the continents showing the large mountain ranges which always have inspired geologists to geodynamic models explaining their origin. Elevations higher than 1 000 m above sea level are shown as grey, elevations higher than 3 000 m are shown in black. The map bears all the characteristics of a good model. It captures the essence of the planets distribution of high mountain ranges within only about 0.01 m^2 of this page, which is only possible by making a range of simplifying assumptions, for example on resolution or choice of cut-off elevation. As such, elevations above 3 000 m in the Azores or Japan escape the scale. Also – while the model largely succeeds in its aim – it is also "wrong" in some places. For example, much of the land surface of (ice covered) Greenland and Antarctica is actually below sea level

forces or velocities which cannot be observed directly because of their enormous time scale or depth.

If the choice of parameters that we consider in our model (and the rules that relate them to each other) are good, then our model is good and it will predict many new observations which will be proven to be correct by future observations. If our choice of parameters and rules is bad, then our model may explain the one or other field observations, but it will predict many other features that will be proven wrong by future observations. Modeling is therefore an iterative back and forth between the choice of parameters and rules that are to be considered or neglected, new observations in nature and finally improvement of the model based on the new observations. Good models are *consistent* with a large number of observations (s. p. 5), but models are hardly ever *unique* in fitting those observations.

Consistent versus Unique. The difference between consistent and unique is an important one that is often not recognized, even by modelers them-

selves. *Unique* means - as the word says - that the model is the very only explanation for a set of observations. *Consistent* means that the proposed model *does* explain a set of observations, but that other models may also explain the same set of observations. The largest majority of models are consistent but by no means unique. For example, the heliocentric Copernican model for the solar system (which states that the planets rotate around the sun as initially suggested by Aristarchus of Samos 310 - 230 B.C., but only brought to wide acceptance by Corpernicus in the early 16th century) is a model which is consistent with our observations of when the sun rises and sets and so on. However, a *geocentric* model (initially designed by Ptolemy) in which the planets and the sun rotate around the earth is possible too. The geocentric model is amazingly more complicated than the heliocentric model and it involves weird planetary motions including epicycles and cycloid curves. However, it also is consistent with our observations on planetary motions. Neither the heliocentric, nor the geocentric model are therefore unique. When viciously defending a model in a discussion it is always sobering to remind oneself that practically all models are only consistent (at best).

Good versus Accurate. The difference between *good* (or possibly: "adequate") and *accurate* models is related to the difference between consistent and unique models, but it is not quite the same. Here, it is important to realize that the *best* model must not be the most *accurate* model! The best model is the one that finds the best balance between accurate description and simplicity. A good example for this is given by a comparison of Newton's law of gravitation and relativity theory when applied to the description of planetary motion (e. g. Hawkins 1988). Newtonian theory states that the gravitational attraction, F, between two masses is directly proportional to the masses of the two bodies m_1 and m_2, and inverse proportional to the square of the distance r between the two bodies. This model is incredibly simple and may be described by a simple equation:

$$F = G\frac{m_1 m_2}{r^2} \quad . \tag{1.1}$$

The constant of proportionality is called the gravitational constant, G. Its value may be found in Table D.4 in the Appendix of this book. This model describes the elliptic motions of the planets (that were discovered by Kepler in order to improve the Copernican model) extremely well. However, very detailed measurements early this century showed that the motions of some planets differ a bit from those described by eq. 1.1. These differences may be explained with the model of general relativity, which describes the planetary motions more exactly than Newton's law. Indeed, general relativity shows that many concepts of Newtonian physics are "wrong", for example the constancy of mass (general relativity states that mass depends on speed). Thus, one might consider Newton's model to be superseded by general relativity and use this new model from now on.

However, general relativity is much harder to grasp than the intuitive understanding of eq. 1.1. It is therefore often not very practical to use. In fact, for the largest majority of purposes – for example to find a planet with a telescope in the sky, or for the interpretation of gravity anomalies by a geologist – Newton's model (eq. 1.1) is sufficient. Thus, for most purposes Newton's model is better (because simpler), albeit less accurate. In short, a good model should find a good balance between *simplicity* of the model and *accuracy* in describing a set of observations and this balance depends on the problem to be solved.

Accuracy versus Precision. In the last paragraph we have used the word *correct* to describe a very good correlation between model description and observation in nature. In general this is the same what is meant by the word *accurate*. However, *precision* is something different. Precision describes how good a model or an experiment can be repeated with the same result (s. p. 419). Let us illustrate this with an example. A radiochemical analysis may indicate that a rock formed 100 my ago. This analysis is very precise if every time we perform it, we arrive at the same age of 100 my. This applies to errors as well. The analysis is still called very precise if we come up with an answer of 100± 50 my, if that answer is reproducible with the identical error limits and we know these error bars very well. However, the radiometric age above is not very accurate. In fact, even analyses with very small analytical errors may be not very accurate at all. It could be that a precise but inaccurate age of 100± 0.1 my was obtained for a rock that actually formed 150 my ago, which in itself bears important information, for example, why the chosen radiometric system re-equilibrated 50 my later.

In conclusion of this section, let us define a good model. A model is a description of nature that has the three following properties:

- It should describe a *large* set of observations with a comparably *small* set of parameters.
- A good model must be useable as a tool to make predictions about fact that have not been observed yet.
- It must be possible to test a good model by making new experiments or observations.

Note that none of these three requirements includes *accuracy*. The deciding factor for a good model remains the balance between accuracy and simplicity. Accurate description of nature is a virtue that remains reserved to (explanation-free) collections of measured accurate data. (Although, remember that many measured data may not be accurate either.)

Geodynamics describes the dynamic evolution of earth through space and time. This dynamic evolution occurs on time scales of up to many hundreds of million years and spatial scales of up to thousands of kilometers. Direct observation is therefore often difficult. Geodynamics is therefore a science which

relies to a much larger extent on the model tool than many other sciences. Mapping in the field, analogue modeling in the laboratory and programming on the computer are all three modeling techniques that are of equal importance in this process. Integrated use of various modeling techniques is the most elegant way to arrive efficiently at a good description of the nature around us.

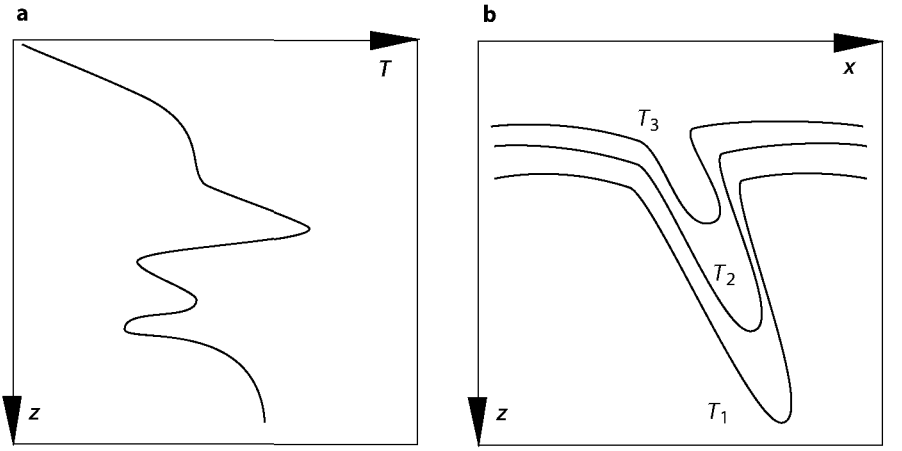


Figure 1.2. Examples of illustrations of one- and two-dimensional models. **a** shows an example of temperature evaluated as a function of a single spatial coordinate, z. The axes of the surface of this page are used up for this one-dimensional description. The strange geotherm is uniquely defined: Every depth corresponds uniquely to a single temperature. **b** shows a two-dimensional model, for example temperatures in a subduction zone as a function of depth z and horizontal distance x. The surface of this page is used up for the two spatial dimensions of the model and the evaluated variable must be portrayed by the contours

1.2 Spatial Dimension of Geological Problems

When a geological process is to be described by a simple model, one of the first decisions that has to be made is often the number of spatial dimensions that are to be considered. This decision has to be made according to the requirements of a good model, as discussed above. That is, it should be tried to describe the problem in question with as few spatial dimensions as possible, without loosing the essence of the problem. Fortunately, a very large number of geological problems can be considered one-dimensionally or even without any spatial dimension. Before we go on to discuss some problems that require one-, two-, or three-dimensional consideration, it should be emphasized that "dimension" need not always be the "spatial dimension". For many problems

it is important to consider time or some other independent variable as the dimension which may then become an axis on a diagram which we use to illustrate some feature of our description. In this book we will stick to (unless otherwise noted) the meaning of a *spatial* dimension when we talk of the dimension of a problem and use the SI *units*, when we use another variable as our "dimension".

One-dimensional Problems. A simple example of a one-dimensional problem is the description of temperature in the lithosphere as a function of depth (Fig. 1.2a). A one-dimensional description of this problem is sufficient when the lateral extent of the plate is large compared to the thickness of the lithosphere and there is no other lateral changes in physical parameters or structure. Continental lithosphere is often of the order of 100 km thick, but continents are usually many hundreds or even thousands of kilometers in lateral extent. The temperature as a function of depth can therefore be well-illustrated in a diagram in which one axis denotes depth and the other temperature. The depth axis is the dimension of the problem, the temperature axis is the evaluated variable. Similar logic may be applied to the description of temperatures around magmatic dikes.

One-dimensional problems need not be in Cartesian coordinates, but can also occur in spherical or cylindrical coordinates, for example temperatures around a spherical intrusion or compositional zoning profiles in a cylindrical crystal. One-dimensional descriptions are useful when there is little or no variation of the variable of interest in the other spatial directions.

Two-dimensional Problems. An example of a typical two-dimensional problem is the temperature distribution in *subduction zones*. There, the subduction angle as well as the plate thickness are both critical for the shape of the isotherms and a vertical (z) *plus* a horizontal spatial coordinate (x) are therefore needed for a meaningful description (Fig. 1.2b). Fortunately, the third spatial dimension – the direction parallel to the trench – can often be neglected because trenches are usually long and the subduction direction is often at roughly right angles to the trench. Calculated temperatures can be illustrated as contours on a diagram with two spatial dimension of the axes.

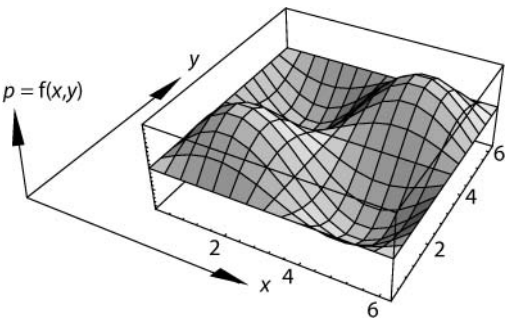


Figure 1.3. A "three-dimensional" illustration of the two-dimensional function $p = \sin(x) \times \sin(y)$ shown on the two-dimensional paper of this book page

The same two-dimensional problem can also be illustrated by using a perspective sketch where the two spatial coordinates *and* the temperature axis are all shown in one diagram. Fig. 1.3 illustrates such a "three-dimensional" illustration of a two-dimensional model using the abstract function $f(x, y) = \sin(x) \times \sin(y)$ as an example. "Three-dimensional" is shown here in quotation marks, firstly because the third dimension (p, or T in Fig. 1.2b) is not really a spatial dimension but the evaluated variable, and secondly because a perspective sketch is still drawn on the two dimensions of this page. Thus, such drawings remain two-dimensional on paper (unless they are built of wire or something else) and they only appeal to the three dimensional imagination of the reader. For two-dimensional models that are used to evaluate temperatures "three-dimensional" illustrations are not common practice, but for geomorphological models and many geophysical questions "three-dimensional" illustrations of two-dimensional model results are quite instructive and are often used.

However, be careful! The fact that two-dimensional models are often shown as surfaces over a grid of two spatial coordinates should not be mistaken for a real three-dimensional model. The third dimension is only the evaluated variable! For Fig. 1.2b we could write T=f(x, z): temperature is a function of x and z only. In landscape models (e. g. Fig. 1.4) this is often quite confusing as the evaluated variable (surface elevation) has the same units (meters) as the two spatial coordinates on which it was evaluated. To stop confusion it is often useful to use the expression *potential surface*. Just like geophysicists evaluate gravitational- or electromagnetic- potential as a function of two spatial coordinates, so can surface elevation or temperature be viewed as a *potential surface* overlying the plane defined by the two spatial coordinates.

Three-dimensional Problems. Three-dimensional models are not only very difficult to design, their results are also hard to understand, to explain or to show graphically. For many geological problems – for example those that involve the modeling of stress or strain – three-dimensional models involve tensors algebra, which makes it often hard to follow their results intuitively. Three-dimensional models should therefore only be used if the problem that is to be solved cannot be simplified in its spatial dimensions (s. p. 7). Examples of important geological problems that are inherently three-dimensional are mantle convection or oblique subduction. Such problems are only solvable with three dimensional models and brave earth scientists use modern methods of calculation and illustration to tackle such difficult problems (e. g. Braun and Beaumont 1995; Platt 1993a). Practically, three-dimensional model results can only be illustrated on a computer screen or with series of contoured diagrams or videos.

Zero-dimensional Problems. A very large number of geological problems can be solved quite elegantly without considering any spatial dimensions. For example, the surface elevation of mountain belts in isostatic equilibrium or the influence of heat production on temperature of rocks can be estimated

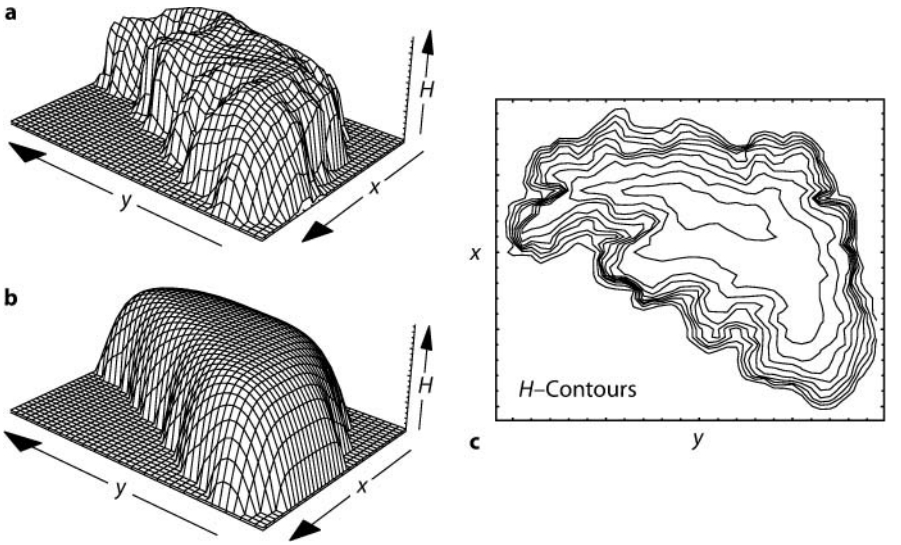


Figure 1.4. **a**, **b** "Three-dimensional" illustration of a two-dimensional model shown on the two dimensions of this page. **c** Two-dimensional illustration of the same model results. The coordinates x and y are the spatial coordinates of the model and H is the evaluated variable - or potential surface. In other models the vertical axis may be the gravitational potential (in m s^{-2}), temperature of a metamorphic terrain or the concentration of an element in a crystal. However, in the shown example, H is of the same units as the spatial coordinates - the model is a landscape model. **a** and **c** are illustrations of field data of the shape of Ayers Rock in Central Australia and **b** shows the modeled shape according to Stüwe (1994)

without spatial considerations and can still give use enormous insight into the nature of many tectonic processes. When neglecting all spatial dependence of a problem, this gives us the great freedom to evaluate the model results graphically as a function of two variables simultaneously. Both axes that may be drawn on paper can be used to evaluate the influence of two different variables against each other.

Zero-dimensional problems should not be confused with *dimensionless variables* or *normalized variables*. In many problems of this book we will encounter variables that are normalized to some standard value and therefore have no units. This is called a *dimensionless variable*. Dimensionless variables are great to get a feeling what a certain number means in relationship to some better known value. For example, the elevation of Mt Blanc, $H_{(MtBlanc)}$, can be either given in meters, or it could be described in terms of its proportional elevation h (in %) relative to, say, Mt Everest:

$$h = \frac{H_{(\mathrm{MtBlanc})}}{H_{(\mathrm{MtEverest})}} = \frac{4\,807\,\mathrm{m}}{8\,848\,\mathrm{m}} = 0.543 \quad . \tag{1.2}$$

$h = 0.543$ is the dimensionless elevation of Mt Blanc relative to Mt Everest. The use of the dimensionless elevation in eq. 1.2 may not be immediately clear. The next example will illustrate the usefulness of this approach better.

When describing the thermal evolution of contact metamorphic aureoles, (sect. 3.6) we will often encounter dimensionless temperatures of the following form:

$$\theta = (T - T_b)/(T_i - T_b) \tag{1.3}$$

There, T_i and T_b are the temperatures of an intrusion and the background temperature of the host rock, for example: $T_i = 900\,°C$ and $T_b = 300\,°C$. T is the variable temperature which may change as a function of distance from the intrusion or time. When $T = 600\,°C$, then this in itself is not very instructive. However, when expressed as θ=0.5 we can see that this temperature is exactly half way between the host rock and the intrusion temperature. For many questions this is much more instructive.

In other examples we will encounter even more complicated formulations of dimensionless variables. For example, in diffusion problems it is often useful to evaluate temperature as a function of the dimensionless variable:

$$T = \mathrm{f}\left(\frac{\kappa t}{l^2}\right) \tag{1.4}$$

This may appear quite confusing, but it has also the purpose of simplifying the results, similar to what we did in eq. 1.3. Eq. 1.4 shows that the variables κ (diffusivity), t (time) and l (size) are coupled in the particular form of this equation ("f()" in eq. 1.4 means "function of"). Using dimensionless variables is not only useful for the better illustration of the meaning of a result, but is also a great aid in differential calculus.

1.2.1 Reducing Spatial Dimensions

Deformation of lithospheric plates is – in the most general case – a three-dimensional problem (however: s. sect. 2.2.2). In three dimensions, stress, strain and strain rate are described by tensors and any fully three-dimensional description of continental deformation does therefore involve tensor calculations (s. sect. 5.1 and B.3). In order to reduce the complications that arise in such calculations, it is useful to see if the number of considered dimensions can be reduced. When describing lithospheric deformation, there are various well-established simplifications that allow us to neglect some or even all of the components of a tensor. Of course, whether or not these simplifications should be used depends on the nature of the problem. Two of these simplifications are important enough for plate tectonic modeling so that they are mentioned already in this first chapter of the book.

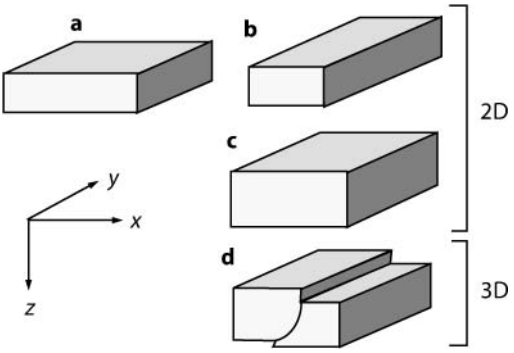


Figure 1.5. Illustration of two- and three-dimensional deformation models (abbreviated with 2D and 3D in the figure). The shortening of the block shown in **a** in the x direction is compensated in **b** only by stretching in the y direction. The thickness of the block remains constant. This is plane strain deformation. In **c** the shortening of the block is compensated by both stretching in the y and z directions. The thickening or stretching in the z direction is homogeneous. This corresponds to the two-dimensional thin sheet approximation. In **d** the shortening in the x direction is compensated by stretching in the y direction and also by inhomogeneous thickening in the z direction. This kind of deformation can only be described with a fully three-dimensional model. In **b** and **c** the stretching in the y and z directions is no function of x (in contrast to Fig. 1.6). This is no requirement of the plane strain or thin sheet approximations. The illustrated geometries are therefore special cases which could also be described one-dimensionally. Stretching in y and z direction could both be evaluated as a function of the shortening in x only

Plane Strain Approximation. The plane strain approximation helps to reduce three-dimensional problems to two dimensions. It assumes that all deformation is strictly two-dimensional so that all strain and displacement occurs in plane and no strain perpendicular to this plane. In plate tectonic modeling we normally consider only the case of no volume change. Then, the total amount of shortening in one spatial direction must be compensated by stretching in the other (Fig. 1.5b; Fig. 1.6). No area change occurs. Tapponier (e. g. 1982) has made great advances in our understanding of continental deformation using this assumption in his descriptions of the India-Asia collision. Plane strain deformation may be viewed as the deformation of a thin film of material that deforms between two fixed parallel plates. It is not *plane stress*, as the normal stresses on the surfaces of the confining plates will vary, depending on where deformation concentrates. Plane strain modeling is a good approximation when the lateral extent of deformation is much larger than the extent in the direction normal to the two modeled dimensions, e.g. crustal thickening or thinning when modeling continental deformation in plan view.

Thin Sheet Approximation. When lithospheric shortening in one horizontal direction is compensated both by stretching in the second horizontal direction and by thickening (or thinning) in the vertical, the lithospheric deformation becomes three-dimensional. However, such scenarios can still

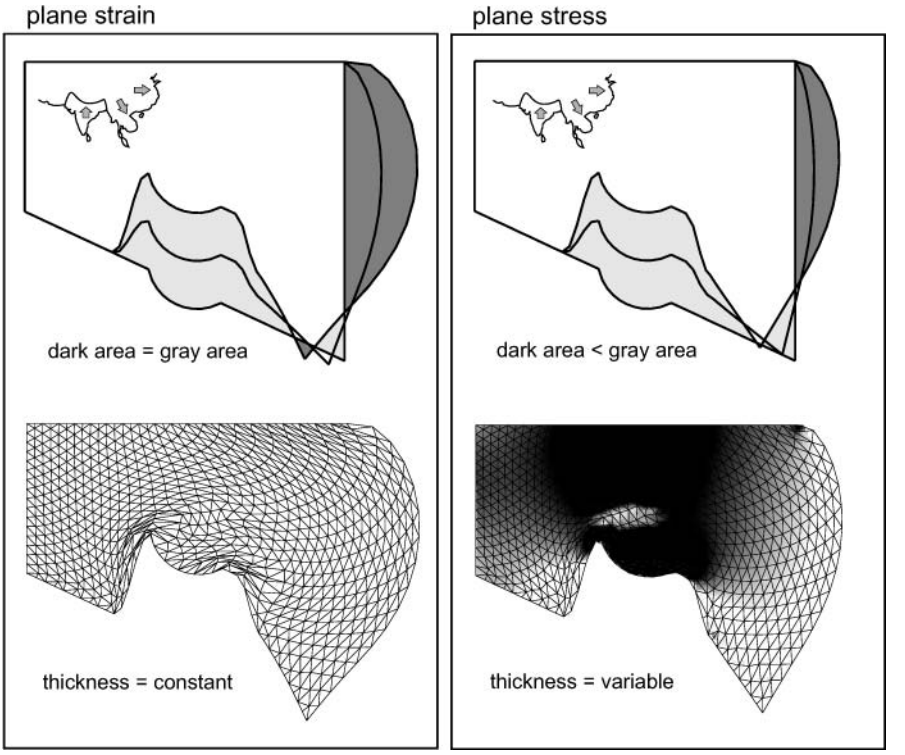


Figure 1.6. A comparison of modeling in two dimensions assuming plane strain (left) and plane stress or a thin sheet (right hand row). The top two diagrams show the outline of the model region at three different time steps of the runs. The bottom two diagrams show the mesh at the final time step. The shading indicates plate thickness. The initial geometry and boundary conditions are chosen to resemble the India-Asia collision zone (shown as inset at the top) (Houseman and England 1996). In plane strain, the area of the plate remains constant throughout deformation: The light shaded area is identical in size to the dark shaded area. The area loss in the right hand row is compensated by thickening in the third dimension. The boundaries along the top and left hand sides are force to remain fixed. The right hand boundary is stress free in both tangential and normal direction. The bottom boundary is tangentially stress free and is forces to move northwards at the constant rate that ends up in the shape of the plate boundary between Indian and Asian plate. The model region is assumed to have a viscous rheology. Model calculated with the finite element package BASIL of Barr and Houseman (1996)

be modeled in two dimensions assuming the *thin sheet* approximation (s. e.g. Houseman and England 1986a; s. however: Braun 1992). The thin sheet model is based on the assumption that the normal stresses at the surfaces of the plate are constant and that there are no shear stresses on horizontal planes. Thus, the thin sheet approximation is also called the *plane stress* approximation. As a consequence, the plate may thicken or thin in the vertical

direction as to maintain the surface stresses constant but there are no vertical strain rate gradients (Fig. 1.5; 1.6) (England and McKenzie 1982, England and Jackson 1989). Using z for the vertical spatial coordinate and $\dot{\epsilon}$ for strain rate, this can be described by:

$$\frac{\mathrm{d}\dot{\epsilon}}{\mathrm{d}z} = 0 \quad . \tag{1.5}$$

The thin sheet approximation is a good approximation for the description of lithospheric scale deformation when:

1. The shear stresses at the surface and the base of the lithosphere are negligible.
2. If the topographic gradients at these two surfaces are small.

Both are usually given on the scale of whole lithospheric plates (s. more detailed discussion in sect. 5.3 and 6.2). Note however that this description can only describe homogeneous thickening of the lithosphere. Thus, this thickening strain can be evaluated as a variable without the need for a third spatial dimension. Extended formulations have been devised that can help to overcome some of these problems (Medvedev and Podladchikov 1999).